INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS, VOL. 3, 314–315 (1983)

LETTER TO THE EDITOR

In response to the comments by Bruce A. Finlayson on the paper by G. A. Keramidas, 'A Generalized Variational Formulation for convective Heat Transfer', *Int. j. numer. methods fluids*, **1,** 305–322, (1981), the author would like to clarify certain points.

M. A. Biot has made considerable contributions to the field of heat transfer and Lagrangian Thermodynamics.[1,2] His work has been valuable to many researchers and has been discussed and extensively referenced by Keramidas,[3–8] contrary to Finlayson's assertions. Furthermore, journalistic style does not necessitate continually referencing original work. The paper in the *International Journal of Numerical Methods in Fluids* cites Keramidas' previous work which include comprehensive discussions of Biot's formulations. Although Biot's basic definition for the heat flow vector is not the same as Keramidas' definition for the heat displacement vector, this author follows Biot's philosophy in using and extending concepts of classical mechanics. However, certain difficulties that exist in Biot's formulation are due to the way that the heat flow vector is defined. For example, in order to use this vector in deriving the equations of coupled thermoelasticity, a definition different than the one used for deriving the heat equation is necessary. These difficulties and contradictions can be overcome by the definition of the heat displacement introduced by Keramidas,[3,4] thus resulting in a uniform transition from one type of equation to another.

Finlayson refers to variational analysis as being unduly complicated and, therefore, having no advantages.[9] Finlayson continues by asserting that Keramidas has ignored this literature as well. However, to quote from Keramidas.[5]

> Nearly all the published applications employing Biot's method to find approximate solutions have been based on the use of the auxiliary field of the heat flux vector. As pointed out by Finlayson and Scriven,[10] to apply Biot's formulation (in terms of the heat flux vector) involves considerable difficulties when two or three-dimensional problems are considered.
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> Although eqns. (24) and (34) may be regarded as variational principles in a broad sense, it has been pointed out by Finlayson and Scriven[10] that in strict logic they are not variational principles in the sense of minimum and maximum principles. However, they may be considered as alternative formulations of the governing equations which facilitate the application of generalized coordinates.

Unfortunately Finlayson seemed to overlook this reference in preparing his comments.

Biot's book on *Variational Principles in Heat Transfer* discusses the Galerkin finite element method without derivation. As Biot further suggests in this book and as has been demonstrated by Keramidas,[7,8] the Galerkin method can be derived as a special application of the generalized Lagrangian formulation. As such, the Galerkin's method can be regarded as equivalent to the generalized method. Finlayson seems to overlook this point.

Another point that Finlayson refers to is the choice of the heat displacement vector, over the temperature field. Such a choice not only simplifies the derivations of the finite element method but it also gives approximate solutions which are computationally more efficient.[8] The generalized approach, or Galerkin's method for that matter, can be derived in terms of a set of generalized co-ordinates which represent either of the two variables. The advantage is that one needs to derive only one set of equations in order to solve a problem in terms of heat displacement or temperature. Finlayson correctly recommends comparisons between solutions. This has been done for both the fundamental and complementary variational formulations.[8] Furthermore, in previous studies, extensive comparisons between Biot's defined heat flow vector and the present heat displacement vector have been done.[3–5]

Finally, the author would like to conclude by saying that the generalized approach is admittedly not a fundamentally new method to obtain approximate solutions, but is an extremely useful extension of the principles of classical mechanics. It is hardly constructive to deny the obvious advantages that stem from the use of generalized principles, such as the principle of virtual work or the concept of generalized coordinates.

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ANNOUNCEMENTS

INTERNATIONAL CONFERENCE ON NUMERICAL METHODS IN LAMINAR AND TURBULENT FLOW

The University of Washington, Seattle, U.S.A. 8–11 August 1983

Objectives

The objectives of this conference are similar to those of the first held in Swansea, 1978, and the second, held at Venice, 1981. Again the main objective is to consolidate the recent advances in the application of numerical techniques, particularly finite difference and finite element methods, to solve laminar and turbulent flow problems. Both techniques have received considerable attention in recent years and their application and development is continually expanding. It is hoped that the conference will provide a forum for numerical analysts to present new numerical methods and applications and experimentalists to present a comparison between measured quantities and calculated values using standard numerical techniques. The subject matter should be of interest to both researchers and industry.

Provisional session headings

Laminar Flow
Lubrication
Turbulent Flow
Boundary Layers
Flow with Separation
Estuary and Coastline Hydrodynamics
Flow in Rivers and Channels
Turbo Machinery
Meteorology
Reactor Technology
Free and Forced Convection
Coupled Conduction and Convection
Turbulent Heat Transfer
Explosions
Scientific and Industrial Applications

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